Intermediation and Investment Incentives

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Abstract

Many products and services are not sold on open platforms but on competing for-profit platforms, which charge buyers and sellers for access. What is the effect of for-profit intermediation on seller investment incentives? Investments in cost reduction, quality, or marketing measures are here the joint and coordinated efforts by sellers. As for-profit intermediaries reduce the rents that are available in the market, one might naively suspect that sellers have weaker investment incentives with competing for-profit platforms. However, we show that for-profit intermediation may lead to overinvestment when free access would lead to underinvestment because investment decisions affect the strength of indirect network effects and thus access prices. We characterize the effect of for-profit intermediation on investment incentives depending on the nature of the investment and on which side of the market singlehomes. Our analysis generalizes to non-coordinated seller investments.

Keywords: Two-Sided Markets, Network Effects, Intermediation, Investment Incentives

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1 Introduction

How does the market environment affect manufacturers’ investment incentives? In general, it is well-known that manufacturers may underinvest in technology or marketing because they cannot fully appropriate the surplus that is generated when selling a product. However, little is known about the influence of market microstructure or trading environment on these investment incentives. Addressing this issue is important as we observe that most consumer products are not sold directly but via intermediaries. These intermediaries come in various forms. Retailers may rent shelf space to producers, shopping mall developers rent stores to producers (or franchisees), trade fairs rent booths to exhibitors, internet retailers list products in their virtual shop. In all these market environments, the prices for the goods to be traded are set by the producers and not by the intermediary (or retailer).\(^1\) Intermediaries obtain revenues by charging for access and usage of the platform. This is so for trading platforms but also, for instance, for software platforms, which grant licences to application software developers and charge users for access (by selling the respective operating systems).\(^2\)

In this paper, we analyze this type of market environments and, more precisely, we address the following question: How does for-profit intermediation affect manufacturers’ investment incentives? We provide a theoretical investigation to address this question. The central message of the paper is that a manufacturer’s investment decision is substantially affected if intermediaries charge for access to their platforms. This message is derived in a particular setting that is motivated below.

With the rise of B2B and B2C commerce, the above question has become even more relevant. Intermediaries may become active in different ways. They may fix bid and ask prices and therefore alleviate search inefficiencies, which arise, e.g., under random matching. The presence of a dealer-intermediary can then be seen as an implicit screening device between seller and buyer types (see e.g. Gehrig, 1993, and Spulber, 2003). In many markets, however, search inefficiencies may be so pronounced that buyers and sellers always trade via a platform. This is clearly the case if the platform provides part of a system that complements the product provided by the seller. A good example for this is the video game industry (and other software industries) in which game developers write their applications for particular game platforms.

\(^1\)Hagiu (2007) documents the move towards decentralized pricing in retailing, i.e. manufacturers obtaining control over retail prices. See also Nocke, Peitz, and Stahl (2007).

\(^2\)Such a software platform may be bundled with a hardware platform as in the case of Apple computers, videogame platforms (offered by Sony, Microsoft and Nintendo), and Palm.
In this case, a video game platform aggregates demand and balances the two sides of the market through the use of price instruments (as in the literature on two-sided markets, see e.g. Rochet and Tirole, 2003, and Armstrong, 2006). In such an industry, we can abstract from any search efficiencies but rather focus on indirect network effects that arise due to group size.

We analyze how seller investment incentives are affected by the presence of two competing for-profit platforms. We may call trade taking place through for-profit intermediaries *intermediated trade*. These intermediaries set access or membership fees on both sides of the market. Conversely, in the absence of for-profit platforms that can restrict access and use of the platform, trade is *non-intermediated* or takes place via open trading platforms which can be accessed without charge. We present a stylized model with two-sided indirect network effects on two competing platforms. Participants on both sides of the market choose which platform to visit; we contrast different scenarios according to whether buyers and/or sellers are allowed to trade via both platforms (i.e., to multihome) or are restricted to use a single platform (i.e., to singlehome). We capture size effects in the form of variety seeking buyers who have a downward-sloping demand function for each product that is available. Our benchmark is a market in which buyers and sellers interact through two open platforms, whose access is free of charge.\(^3\) To address the role of (imperfectly) competing intermediaries, we compare investment incentives with two competing for-profit platforms to those with such open platforms.

What do we mean by seller investments? Investments may take the form of cost reduction, quality improvement or marketing measures that facilitate price discrimination or expand demand. In all these cases, we postulate that investments are the joint and coordinated efforts by sellers (in the form of horizontal agreements). For instance, sellers may form cost-reducing or quality improving R&D joint ventures. Participation can be seen as mandatory, if industry associations enforce compliance. Also, minimum quality standards that are advocated by business associations can be interpreted as a coordinated investment decision on the seller side. Business associations may engage in joint marketing activities that expand demand. A case in point are the actions that music labels have taken against piracy, coordinated in the U.S. by the RIAA. Here, incentives are affected by the need to have music distributed by a

\(^3\)For simplicity, our main analysis covers only independent sellers. For extensions see the conclusion and Appendix 1. Under imperfect competition between sellers, buyers benefit from more competition between sellers through lower prices (see Gehrig, 1998, Hagiu, 2006a, and Nocke, Peitz, and Stahl, 2007).
Another example is the “got milk?” U.S. advertising campaign by the Milk Processor Education Program (MilkPEP), which is funded by U.S. milk processors. Similarly, information sharing agreements and joint data collection activities may result in demand expansion and improved price discrimination possibilities at the seller level and thus constitute a coordinated investment decision. Here, not only the total surplus may be affected but the distribution between buyer and seller is affected by the activity. A case in point are industry associations who require their members to share some data. A more recent phenomenon with voluntary participation is so-called database co-ops. To become a member a firm must contribute its own database. As an example, to join the Abacus B2B alliance and make use of its dataset a firm must contribute names that had transactions with this firm (see www.abacus.com for details). Our analysis can be extended to voluntary participation, as we state next.

We show that our results under coordination carry over to an environment with non-coordinated investments by sellers, provided that platforms can price discriminate among investing and non-investing sellers. Here, the platforms’ price discrimination may depend on a particular quality standard to which sellers adhere or a membership. This shows that our framework extends to voluntary participation in trade association, provided that platforms can use this information in their pricing decision. Alternatively, if the investment or membership cannot be directly monitored, platforms may be able to measure the strength of the indirect network effect, which is determined by the investment, and condition their fees on this measure. We also note that there are other markets in which a third party has developed a new technology (or standard) that is available for licensing. Then the coordinated adoption of this technology through licensing can be interpreted as a coordinated investment decision.

Why should the type of platform matter for seller investment incentives? Clearly, the presence of for-profit intermediaries reduces the rents that are available in the market. Therefore, one might naively suspect that sellers have unambiguously weaker investment incentives with intermediated trade. However, this ignores margin effects. Investments affect the size of the network effects and thus competition between intermediaries, which feeds into access fees. In particular, when innovations increase buyer surplus, intermediaries react to the corresponding

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4However, to be in line with our model platforms must decentralize pricing decision to music labels, which is different from the current iTunes business model.

5Such horizontal information sharing agreements have also been analyzed in the banking industry (see e.g. Gehrig and Stenbacka, 2007). Abstracting from rivalry between sellers (banks), our theory points to the role of infomediaries that bring consumers and banks together when e.g. consumers try to find a loan.
investments by lowering access fees on the seller side. As a consequence, sellers internalize changes in buyer surplus if products are traded on for-profit platforms, whereas they do not in the context of open platforms. Then investment incentives can be stronger with competing for-profit platforms than with open platforms. The exact relationship between investment incentives and for-profit intermediation depends on which side of the market singlehomes and on the nature of the investment effort. In our linear specification, we show the following results: (i) when both sides singlehome, trade via for-profit platforms raises seller incentives to innovate in cost-reduction and in quality, but lowers incentives to invest in price discrimination (and the effect depends on parameter values for investments in demand expansion); furthermore, in such a market, a social underinvestment problem with open platforms translates into a social overinvestment problem with proprietary for-profit platforms; (ii) when sellers singlehome and buyers can multihome, trade via for-profit platforms leads to stronger incentives to innovate whatever the nature of the innovation; (iii) when buyers singlehome and sellers can multihome, the exact opposite prevails (i.e. if trade takes place via for-profit platforms incentives to innovate are reduced).

In a recent empirical paper, Boudreau (2006) investigates the effect of the degree of openness of the platform on seller incentives in the computer industry. He finds that restricted access and some control over the platform led to more investments in innovation than highly open strategies by platforms. A potential reason for these findings, as has been recognized by Boudreau (2006, p. 2), are strategic effects: “In that the opening of a system will also surely affect the ‘within-system’ competition (and perhaps even between system strategic interactions), suppliers’ strategic incentives to make investments in innovation might also be affected.” Our paper presents a formal framework to address the issue of competition between platforms.

Our paper connects to the burgeoning recent literature on two-sided markets. Compatible with this literature is the view that intermediaries possess property rights on a platform and thus can make profits from charging access or usage fees on both sides of the market. Seminal contributions in this literature include Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006). Our set-up of the two-sided market borrows from Armstrong’s models with singlehoming and with competitive bottlenecks where two competing intermediaries set access prices on both sides of the market.6 In this model, we provide a micro foundation of seller profits and consumer utilities and analyze sellers’

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6The particular setup is even closer to Armstrong and Wright (2007).
investment incentives in comparison to a model in which all sellers and buyers trade via open platforms. While most work has exclusively focused on for-profit platforms, Hagiu (2006a) and Nocke, Peitz, and Stahl (2007) compare for-profit to open platforms. However, all these papers are silent about investment incentives in such markets.

More generally, our paper contributes to the micro-market structure and intermediation literature. Here, an alternative line of research has taken the view that intermediated trade (by dealers who set bid and ask prices) may avoid inefficiencies that arise in random matching environments. In this setting, the coexistence of matching and dealer market leads to a self-selection of types (see Gehrig, 1993). In such an environment, Spulber (2003) analyzes sellers’ (and buyers’) investment incentives. He shows that the introduction of a dealer market in a decentralized matching market leads to stronger investment incentives. Our analysis can be seen as complementary to the work by Spulber (2003) because we abstract from search inefficiencies and rather focus on market size externalities.

Also, our paper borrows from and contributes to the literature on R&D joint ventures and R&D agreements (see Kamien, Muller, and Zang, 1992, and the literature that builds on this). These joint ventures or agreements imply certain investment decisions by the participating firms and innovations are shared among the firms. In particular, research joint ventures pool resources within a common functional area to overcome capacity constraints in research activities, pool resources across functions areas, or develop new products or processes in parallel and then share the innovation. In line with a large part of this literature (***references????) we assume that the sellers choose the investment cooperatively, whereas they choose their subsequent actions to maximize private benefits. The contribution of this paper then is to show that the incentives to form such joint ventures or agreements and the particular policy that is chosen by the firms depend on the underlying market microstructure. In particular, it may be affected by the need to obtain access to one of the competing platforms. Similarly, concerning marketing alliances at the producer level, the intensity of joint marketing activities depend on the underlying market microstructure.

The rest of the paper is organized as follows. In Section 2, we set up the model. We then analyze three particular versions of the model: in Section 3, we assume that both buyers and sellers singlehome; in Section 4, only buyers singlehome while sellers are allowed to multihome (in these two sections, we also show that our insights continue to hold if investment decisions

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7 For a presentation of the Gehrig model, see Spulber (1999). For an extended analysis, see Rust and Hall (2003).
are not coordinated among sellers); in Section 5, the opposite prevails as sellers singlehome while buyers are allowed to multihome. In Section 6, we provide a microfoundation for the generic surplus functions used in the previous sections; thereby, we examine the seller incentives to innovate in cost-reduction, in quality improvement, in price discrimination, and in demand expansion. We conclude and discuss possible extensions in Section 7. Three appendices complement the analysis in the main text.

2 The model

We provide an abstract model of trade on a platform that closely follows the literature on two-sided markets and here, in particular, Armstrong (2006) and Armstrong and Wright (2007). There are two sides of the market, the buyer side and the seller side. Suppose that each side is of mass 1. Buyers and sellers can interact on two platforms, 1 and 2, which are assumed to be located at the extreme points of the unit interval. Buyers and sellers are assumed to be uniformly distributed on the unit interval, which gives their location on the line, and to incur an opportunity cost of visiting a platform that increases linearly in distance at rates \( \tau_b \) and \( \tau_s \), respectively.

In order to analyze how the market microstructure affects the ex ante investment incentives of sellers, we compare two different organizations of the trading platforms: intermediated trade (in which platforms are run by strategic profit-maximizing intermediaries) and non-intermediated trade (in which platforms are open). The latter case can be seen as a natural benchmark because we do not want to introduce other differences with respect to the for-profit duopoly model. The comparison between intermediated and non-intermediated trade gives us an answer to the following thought experiment: what would happen, in terms of incentives to innovate, if platforms were open?

Under both trade patterns, we characterize the subgame-perfect equilibrium of the multi-stage game in which, for given investment levels, agents first choose which platform to join and then interact on the platform(s) of their choice. We then examine how the sellers’ equilibrium payoffs would change as a result of some investment that modifies sellers’ and buyers’ payoff functions in the multi-stage game. This change in the sellers’ equilibrium payoffs measures the sellers’ ex ante incentives to achieve the given investment. The investment choice is analyzed at a stage 0, where sellers cooperatively choose the investment level \( y \).

In the case of intermediated trade, we analyze the following four-stage game. At stage 1,
intermediaries simultaneously set membership fees $M^i_b$, $M^i_s$ on the two sides of the market. At stage 2, sellers and buyers decide which platform to visit. At stage 3, sellers set the price of their goods simultaneously. Finally, at stage 4, buyers make purchasing decisions. In the case of non-intermediated trade, buyers and sellers interact through open platforms to which access is assumed to be free of charge. As the for-profit intermediaries disappear, so does stage 1. We thus solve for subgame-perfect equilibria of the multi-stage game made of stages 2 to 4.

The two games share the following assumptions. First, regarding stage 3, we assume that the sellers’ pricing decisions are independent, so that we do not need to make particular assumptions on the timing decision at this pricing stage (we can think of sellers producing perfectly differentiated varieties). This assumption simplifies the analysis and allows us to focus on comparing investment incentives under intermediated and non-intermediated trade. Moreover, as we show in Appendix 1, our results are robust to introducing competition among sellers.

Regarding stage 4, we assume that a buyer at platform $i$ has a downward-sloping demand function for each product that is traded on this platform. We provide in Section 6 a number of micro models of buyers-seller relationships and analyze particular investment decisions in such settings. In particular, we consider sellers with independent and downward sloping demand who engage in cost-reducing R&D, who invest in quality improvements, or who invest in technologies or marketing efforts which allow them to price discriminate between consumers or to expand demand. As for now, we use a reduced-form representation of buyer-seller interaction. Buyer and seller surplus gross of any opportunity cost of visiting a platform are simply computed as

$$v^i_b = n^i_b u(y) - M^i_b,$$  \hspace{1cm} (1)

\text{and} \hspace{1cm}$$v^i_s = n^i_s \pi(y) - M^i_s,$$  \hspace{1cm} (2)

where $\pi$ is the sellers’ surplus from trade with one buyer, $u$ is the buyers’ surplus from trade with one seller, $n^i_b$ (resp. $n^i_s$) is the number of buyers (resp. sellers) active on platform $i$, and $M^i_b$ and $M^i_s$ are the membership fees set by intermediary $i$.

We conclude the description of the model by introducing a number of definitions that will prove useful to analyze the buyers’ and sellers’ platform choices at stage 2. Let $b_{12}$ and $s_{12}$ denote, respectively, the buyer and the seller who are indifferent between visiting platform 1 and visiting platform 2. Using expressions (1) and (2), along with our definition of the
opportunity cost of visiting a platform, we find:

\[ v_b^1 - \tau_b b_{12} = v_b^2 - \tau_b (1 - b_{12}) \iff b_{12} = \frac{1}{2} + \frac{v_b^1 - v_b^2}{2\tau_b}, \]

\[ v_s^1 - \tau_s s_{12} = v_s^2 - \tau_s (1 - s_{12}) \iff s_{12} = \frac{1}{2} + \frac{v_s^1 - v_s^2}{2\tau_s}. \]

Similarly, let \( b_{10} \) (resp. \( b_{20} \)) denote the buyer who is indifferent between visiting platform 1 (resp. 2) and not visiting any platform (thereby getting a utility of zero):

\[ v_b^1 - \tau_b b_{10} = 0 \iff b_{10} = \frac{v_b^1}{\tau_b}, \]

\[ v_b^2 - \tau_b (1 - b_{20}) = 0 \iff b_{20} = 1 - \frac{v_b^2}{\tau_b}. \]

We proceed in the same way on the seller side by defining the indifferent sellers as:

\[ s_{10} = \frac{v_s^1}{\tau_s}, \text{ and } s_{20} = 1 - \frac{v_s^2}{\tau_s}. \]

In what follows, we assume that participation is sufficiently attractive so that all buyers and sellers participate in the market. More precisely, we require (i) \( 0 < b_{20} < b_{12} < b_{10} < 1 \) and (ii) \( 0 < s_{20} < s_{12} < s_{10} < 1 \). Necessary and sufficient conditions are\(^8\)

\[ v_b^1, v_b^2 > 0 \text{ and } \max \{ v_b^1, v_b^2 \} < \tau_b < v_b^1 + v_b^2, \]  
\[ (3) \]

\[ v_s^1, v_s^2 > 0 \text{ and } \max \{ v_s^1, v_s^2 \} < \tau_s < v_s^1 + v_s^2. \]  
\[ (4) \]

In the next three sections, we contrast investment incentives in intermediated and non-intermediated trade under three different scenarios regarding the agents’ ability to interact simultaneously on more than one platform. We first consider situations where both sides of the market singlehome (Section 3). We turn next to situations where one side of the market singlehomes while multihoming is feasible on the other side of the market (Sections 4 and 5). As we will illustrate in Section 6, the investment level affects the demand curve or the cost function firms face and thus has an impact on \( u(y) \) and \( \pi(y) \) (it may also impact price discrimination possibilities).

\(^8\)We state here the conditions in terms of endogenous variables. Below, we develop them for each case, and express them solely in terms of exogenous variables.
3 Two-sided singlehoming

In this section, both sides of the market are assumed to singlehome. Singlehoming environments in the real world can be motivated by indivisibilities and limited resources, or by contractual restrictions. The former applies to certain real-world market places where buyers and sellers can physically locate in only one of them (flea and farmer markets come to mind). For the latter, we find more examples. For instance, taxi companies in Germany sign exclusive contracts with taxi call centers. There also appears to be little multihoming on the consumer side. Similarly, some employment agencies for temporary work can be characterized by singlehoming on both sides of the market. It is less clear to which extent video game platforms can be approximated by two-sided singlehoming. Some gamers have more than one platform and leading game developers nowadays develop the same game for different platforms.\(^9\) Also, it has been claimed that the market for certain specialized magazines, where the magazine serves as the platform and advertisers and readers constitute the two sides of the market, can be described as a market on which both sides singlehome (see Kaiser and Wright, 2006). In addition, many media markets have the property that content is exclusive. If content providers sell their content to consumers while the media platform only charges for the services it offers, we are also in a situation where both sides singlehome (provided that consumers singlehome in these markets).

3.1 Equilibrium for given investment levels

Our reference point is a situation of non-intermediated trade: two open platforms are located at the extremes of the unit interval (that is, they have the same locations as the for-profit platforms in the alternative environment). In this case, each seller has access to half of the unit mass of consumers and derives net surplus (but gross of transport costs) equal to \(\pi/2\) (supposing that access to each open platform is free).

Now consider the market with two competing for-profit platforms. Under conditions (3) and (4), each side of the market is divided into two groups at stage 2: buyers (resp. sellers) located between 0 and \(b_{12}\) (resp. \(s_{12}\)) visit platform 1 and those located between \(b_{12}\) (resp. \(s_{12}\)) visit platform 2. The remainder of the surplus calculation follows along the lines of Section 2.6.

\(^9\)Hagiu and Lee (2007) state that prominent video game publishers have the vast majority of their hit games present on all major video game consoles. E.g., Evans and Schmalensee (2005) report that recently Electronic Arts, a leading game developer, released its games for the Nintendo, Microsoft, and Sony platforms. However, Clements and Ohashi (2005) report that only 17% of titles in their sample was available on multiple platforms.
It follows that
\[ n^i_b = 1 - n^i_s, \]
and
\[ n^i_s = 1 - n^i_b. \]

Using the expressions for buyer and seller surplus (1) and (2), and the facts that
\[ n^i_j b = 1 - n^i_i b, \]
and
\[ n^i_j s = 1 - n^i_i s, \]
we obtain the following expressions for the numbers of buyers and sellers at the two platforms:

\[ n^i_b = \frac{1}{2} + \frac{v^i_b - v^j_b}{2\tau_b}, \]
\[ n^i_s = \frac{1}{2} + \frac{v^i_s - v^j_s}{2\tau_s}. \]

This shows that for given membership fees of buyers, an additional seller attracts \( u/\tau_b \) additional buyers. Similarly, an additional buyer attracts \( \pi/\tau_s \) additional sellers. Combining these two findings, we see that the indirect network effects on each side of the market are measured by the ratio \( u\pi/\tau_b\tau_s \). If the indirect network effects are too strong, two intermediaries cannot be active. To exclude this possibility, we require that
\[ u\pi/\tau_b\tau_s < 1, \]
or equivalently
\[ \tau_b\tau_s > u\pi, \]
which measure the perceived horizontal differentiation between the two platforms be sufficiently large with respect to the gains from trade \( u \) and \( \pi \). To make sure that the second-order conditions are satisfied in the maximization programs of stage 1, we impose a slightly more restrictive condition, namely
\[ 4\tau_s\tau_b > (u + \pi)^2. \]

We can then solve the above implicit expressions for the number of buyers and sellers to obtain the following formulas:

\[ n^i_b = \frac{1}{2} + \frac{u(M^j_s - M^j_b) + \tau_s(M^j_s - M^j_b)}{2(\tau_b\tau_s - u\pi)}, \]
\[ n^i_s = \frac{1}{2} + \frac{\pi(M^j_s - M^j_b) + \tau_b(M^j_s - M^j_b)}{2(\tau_b\tau_s - u\pi)}. \]

The number of buyers at one platform is not only decreasing in the membership fee for buyers on this platform but also, due to indirect network effects, in the membership fee for sellers. If the fees set by the intermediaries are such that \( 0 < n^i_b, n^i_s < 1 \) (which will indeed be the case), expressions (5) and (6) define a unique and stable equilibrium for stage 2 (see Appendix 2 for a proof).

\[ ^{10} \text{In deriving equilibrium membership fees, the analysis follows Armstrong (2006), which also contains an insightful discussion of the effect of platform competition on pricing.} \]
Let us next turn to the first stage of the game at which platforms set prices (for given sellers’ investment levels). Assuming that the intermediary’s cost per buyer is $C_b$ and per seller is $C_s$, we can write platform $i$’s profit as

$$\Pi_i = (M_i^b - C_b) \left( \frac{1}{2} + \frac{u(M_i^j - M_i^s) + \tau_s(M_i^j - M_i^s)}{2(\tau_b \tau_s - u \pi)} \right)$$

$$+ (M_i^s - C_s) \left( \frac{1}{2} + \frac{\pi(M_i^j - M_i^s) + \tau_b(M_i^j - M_i^s)}{2(\tau_b \tau_s - u \pi)} \right).$$

The two intermediaries simultaneously choose membership fees on both sides of the market. First-order conditions of profit maximization in a symmetric equilibrium, i.e. $M_1^b = M_2^b \equiv M_b$ and $M_1^s = M_2^s \equiv M_s$, can be written as

$$M_b = C_b + \tau_b - \frac{\pi}{\tau_s}(u + M_s - C_s),$$

$$M_s = C_s + \tau_s - \frac{u}{\tau_b}(\pi + M_b - C_b).$$

Equilibrium prices on the seller side are equal to marginal costs plus the product differentiation term as in the standard Hotelling model, adjusted downward by the term $\frac{u}{\tau_b}(\pi + M_b - C_b)$. Recall that each additional seller attracts $u/\tau_b$ additional buyers. These additional buyers allow the intermediary to extract $\pi$ per seller without affecting the sellers’ surplus. In addition, each of the additional $u/\tau_b$ buyers gives a profit of $M_b - C_b$ to the seller. Thus $\frac{u}{\tau_b}(\pi + M_b - C_b)$ represents the value of an additional buyer to the intermediary. The higher this value, the more aggressive the price setting among intermediaries on the seller side.

Solving for the Nash equilibrium membership fees, one finds

$$M_b^* = C_b + \tau_b - \pi,$$

$$M_s^* = C_s + \tau_s - u.$$

Sellers’ membership or access fees are lower if there are larger gains from trade on the buyer side. It follows that at equilibrium, $n_1^b = n_2^b = 1/2$ and $n_1^s = n_2^s = 1/2$, so that the equilibrium net surplus of sellers and buyers (gross of transportation cost) are equal to:\footnote{We still need to check that conditions (3) and (4) are met. This is so provided that $\frac{1}{2} \left( \frac{1}{2} u + \pi - C_s \right) < \tau_b < \frac{1}{2} \left( \frac{1}{2} u + \pi - C_b \right)$ and $\frac{1}{2} \left( \frac{1}{2} \pi + u - C_s \right) < \tau_s < \frac{1}{2} \left( \frac{1}{2} \pi + u - C_b \right).$}

$$v_b^* = \frac{1}{2} u + \pi - (C_b + \tau_b),$$

$$v_s^* = \frac{1}{2} \pi + u - (C_s + \tau_s).$$

(7)
We observe that $v_b$ and $v_s$ are increasing in the gains from trade that accrue the other side of the market and, to a lesser extent, also in the gains from trade on the own side.\(^{12}\)

The intermediaries’ equilibrium profits are

$$\Pi^i = \frac{1}{2} (\tau_s - u) + \frac{1}{2} (\tau_b - \pi) = \frac{1}{2} (\tau_s + \tau_b) - \frac{1}{2} (u + \pi).$$

Note that only the joint net gain from trade by buyers and sellers determines each intermediary’s profit. Thus, the distribution of net gains among sellers and buyers does not affect its profit. Furthermore, this profit is decreasing in $(u + \pi)$; i.e., in markets in which gains from trade are high, the intermediary’s profits are low. This result may seem counterintuitive but can be explained as follows. Net gains $u$ and $\pi$ determine the strength of network effects in the industry. If $u + \pi$ is large, this means that additional buyers and sellers are very valuable for intermediaries. Therefore, they compete more aggressively in the market place. If network effects were too strong, only one intermediary would be viable. We restrict attention to situations in which two intermediaries are viable so that $\Pi^i > 0$.\(^{13}\)

### 3.2 Seller incentives to innovate

In terms of investment decisions, we assume for the moment that all sellers make the same investment. That is, investment decisions are coordinated at the level of all sellers in the market; in the next subsection, we discuss how the analysis can be extended to investment decisions that are coordinated at the level of smaller groups of sellers, or are decided even at the individual level.

Suppose that sellers jointly determine their efforts in some R&D or marketing activity that affects some parameter $y$ that enters the profit function of each firm. We can then write maximal profit as a function $\pi(y)$.\(^{14}\) Note that a change in $y$ typically affects buyers’ surplus as well. Therefore, we write surplus as a function of $y$, i.e. $u(y)$. The incentives to innovate are then determined by the effect of $y$ on $\pi$ and $u$.

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\(^{12}\)Note that, in a bargaining environment, this implies that sellers would rather benefit from less bargaining power in a buyer-seller relationship. This may sound counterintuitive. Clearly, in each buyer-seller relationship, a seller benefits from an increase in his bargaining power, everything else equal. However, in the intermediated market, the shift in bargaining power affects membership fees. Taking these effects into account, a seller would like to be weak in the bargaining process.

\(^{13}\)This is the case under the assumption we made above, according to which $4\tau_s \tau_b > (u + \pi)^2$, which implies that platform are sufficiently differentiated.

\(^{14}\)We provide micro foundations for this profit function in Section 6.
How much is a seller willing to pay to acquire this innovation? Consider an innovation from \( y \) to \( y' \). In the *intermediated trade*, sellers are willing to pay up to the increase in their net surplus: \( \Delta^m \equiv v_s(y') - v_s(y) \). From (7), we obtain

\[
\Delta^m = \frac{1}{2} \left[ \pi(y') - \pi(y) \right] + \left[ u(y') - u(y) \right].
\]

In the *non-intermediated trade*, each seller is willing to pay up to the increase in its profit for an innovation from \( y \) to \( y' \):

\[
\Delta^n \equiv \frac{1}{2} \left[ \pi(y') - \pi(y) \right].
\]

Comparing the expressions for \( \Delta^n \) and \( \Delta^m \), we have the following simple result.

**Proposition 1** *In the two-sided singlehoming model, for-profit trading platforms give stronger incentives for sellers to innovate if and only if the buyers’ surplus increases, i.e.,*

\[
\Delta^m > \Delta^n \iff u(y') - u(y) > 0.
\]

To understand this condition, recall that in the intermediated trade, a seller’s net surplus is equal to \( v_s(y) = n_s^* \pi - M_s^* = \frac{1}{2} \pi(y) - [C_s + \tau_s - u(y)] \). If the investment increases the buyers’ surplus, then for-profit platforms will charge a lower fee to sellers, providing an extra incentive to innovate compared to open platforms (where this price effect is absent); naturally, the opposite prevails when the investment decreases the buyers’ surplus.

In Section 6, we will have a closer look at the microstructure of the buyer-seller relationship and the nature of seller investments. Note that since \( n_i^* = 1/2 \) under both types of platform organization, the ranking of per-seller incentives implies the same order of total investment (i.e., the sum of sellers’ willingness to pay for the innovation). While the precise form of condition (8) is an artifact of the linear structure, a general feature is that a higher buyer surplus is desirable for sellers on a for-profit platform but irrelevant on an open platform. Hence, under for-profit platforms, sellers partly internalize improvements of buyer surplus when making their investment decision.\(^{15}\)

**Remark 1** *In a second-best world (where the social planner only decides about the investment level and trade takes place on both platforms), total surplus is maximized with the innovation if*

\[
\frac{1}{2} \left[ \pi(y') - \pi(y) \right] + \frac{1}{2} \left[ u(y') - u(y) \right] > K(y') - K(y),
\]

\(^{15}\)We could then even observe types of investment that *decrease* the seller surplus per buyer \( \pi \), provided that this decrease is more than compensated by an increase in the buyer surplus per seller \( u \). By contrast, sellers would never choose to make such investments if platforms were open.
where $K(y') - K(y)$ is the social cost to carry out the innovation. In the differentiable case, the planner solves $[\pi'(y) + u'(y)]/2 = K'(y)$. Sellers in a market with two open platforms (which simply charge zero access prices) choose an investment level that solves $\pi'(y)/2 = K'(y)$; and sellers in a market with two proprietary platforms choose an investment level that solves $\pi'(y)/2 + u'(y) = K'(y)$. If $u$ is strictly increasing in $y$, there is social underinvestment on open platforms and social overinvestment on for-profit platforms. Reversely, if $u$ is strictly decreasing in $y$, there is social overinvestment on open platforms and social underinvestment on for-profit platforms.

The latter remark implies that public policy steered towards private investments should critically depend on the prevailing intermediation structure. For instance, while the analysis under open platforms suggests that an R&D subsidy may be an appropriate remedy to the social underinvestment problem that prevails in the context of R&D investments (see Section 6), such a subsidy may worsen the overinvestment in a market with for-profit platforms.

### 3.3 Decentralized investment decisions

In the previous analysis, we simplified the investment decisions by assuming that all sellers were agreeing upon the same investment level. Here, we show that our result about the comparison between intermediated and non-intermediated trade still holds if we relax the assumption of coordinated investment decisions.

We start from a situation where a fraction $\mu$ of sellers have invested; these sellers are drawn independently from the heterogeneous population of sellers. We denote by $\pi' \equiv \pi(y')$ and $\pi \equiv \pi(y)$ the net gain from trade for, respectively, a seller that has invested and a seller that has not. Similarly, the net gain from trade for a buyer is $u' \equiv u(y')$ if the buyer interacts with a seller who has invested, and $u \equiv u(y)$ if the buyer interacts with a seller who has not invested. Accordingly, seller and buyer surplus gross of any opportunity cost of visiting a platform $i$ are now written as

$$
\nu^i_s = \begin{cases} 
  n^i_s \pi' - M^i_s & \text{if seller has invested}, \\
  n^i_s \pi - M^i_s & \text{if seller has not invested}, 
\end{cases}
$$

$$
\nu^i_b = n^i_b (\mu u' + (1 - \mu) u) - M^i_b.
$$

We consider two scenarios. In the first scenario, platforms cannot tell investing and non-investing sellers apart; they are therefore forced to set a uniform membership fee to all sellers:
\( M_{s}'' = M_{s}' \). In the second scenario, platforms are able to observe investment decisions and to price discriminate between investing and non-investing sellers: \( M_{s}'' \neq M_{s}' \).

We start by deriving the number of buyers and sellers going to each platform. The indifferent buyer is identified by \( b_{12} \) such that
\[
v_{1} - \tau_{0} b_{12} = v_{2} - \tau_{0} (1 - b_{12});
\]
which is equivalent to
\[
b_{12} = \frac{1}{2} + \frac{(n_{s}^{i} - n_{s}^{2}) (\mu u' + (1 - \mu) u) + M_{b}^{2} - M_{b}^{1}}{2 \tau_{b}}.
\] (9)

The indifferent investing and non-investing sellers are respectively identified by \( s_{12}' \) such that
\[
n_{s}^{b} - \tau_{s} s_{12}' = n_{b}^{2} - \tau_{s} (1 - s_{12}') \]
and \( s_{12} \) such that
\[
n_{s}^{b} - \tau_{s} s_{12} = n_{b}^{2} - \tau_{s} (1 - s_{12}).
\] (10)
(11)

Two-sided singlehoming and full participation imply the following:
\[
\begin{align*}
n_{b}^{1} &= b_{12} \\
n_{b}^{2} &= 1 - b_{12} \\
n_{s}^{1} &= \mu s_{12}' + (1 - \mu) s_{12} \\
n_{s}^{2} &= \mu (1 - s_{12}') + (1 - \mu) (1 - s_{12}).
\end{align*}
\] (12)

### 3.3.1 Uniform sellers’ membership fee

We suppose here that \( M_{s}'' = M_{s}' \). We proceed as above by first solving the system of equations (9) to (12) to find the equilibrium values of \( n_{b}^{i} \) and \( n_{s}^{i} \), expressed as functions of the membership fees set by the two platforms. Then, one can compute platform \( i \)'s first-order conditions for profit maximization:

\[
\begin{align*}
\frac{d}{dM_{b}^{i}} \left[ n_{b}^{i} (M_{b}^{i} - C_{b}) + n_{s}^{i} (M_{s}^{i} - C_{s}) \right] &= 0, \\
\frac{d}{dM_{s}^{i}} \left[ n_{b}^{i} (M_{b}^{i} - C_{b}) + n_{s}^{i} (M_{s}^{i} - C_{s}) \right] &= 0.
\end{align*}
\]

Invoking symmetry, one finds the optimal fees for the two platforms:\(^{16}\)

\[
\begin{align*}
M_{b}^{1*} &= C_{b} + \tau_{b} - \pi - \mu (\pi' - \pi) \left( 1 + \frac{(1 - \mu) (u' - u)}{\tau_{s}} \right), \\
M_{s}^{1*} &= C_{s} + \tau_{s} - \mu u' - (1 - \mu) u.
\end{align*}
\]

\(^{16}\)The details of the computations are in a technical appendix that can be obtained from the authors upon request.
We recover the two extreme cases that we considered before by supposing either that all sellers have invested \( \mu = 1 \), implying that \( M_i^* = C_b + \tau_b - \pi' \) and \( M_i^* = C_s + \tau_s - u' \), or that no seller has invested \( \mu = 0 \), implying that \( M_i^* = C_b + \tau_b - \pi \) and \( M_i^* = C_s + \tau_s - u \).

Because platforms set the same fees, it follows that \( n_1^b = n_2^b = n_1^s = n_2^s = \frac{1}{2} \). We can then compute the buyer equilibrium surplus and the seller equilibrium surplus (gross of opportunity and potential investment costs):

\[
v_s^* = \begin{cases} 
\frac{1}{2} \pi' + \mu u' + (1 - \mu) u - (C_s + \tau_s) & \text{if the seller has invested,} \\
\frac{1}{2} \pi + \mu u' + (1 - \mu) u - (C_s + \tau_s) & \text{if the seller has not invested,}
\end{cases}
\]

\[
v_b^* = \frac{1}{2} (\mu u' + (1 - \mu) u) + \pi + \mu (\pi' - \pi) \left( 1 + \frac{(1 - \mu) (u' - u)}{\tau_s} \right) - (C_b + \tau_b).
\]

If we had free platforms instead of for-profit platforms, we would have:

\[
v_s^n = \begin{cases} 
\frac{1}{2} \pi' & \text{if the seller has invested,} \\
\frac{1}{2} \pi & \text{if the seller has not invested,}
\end{cases}
\]

\[
v_b^n = \frac{1}{2} (\mu u' + (1 - \mu) u).
\]

Let us now compute incentives to innovate. With free platforms, it is simply \( \Delta^n = \frac{1}{2} (\pi' - \pi) \). With for profit platforms, let us look at a firm that was not investing beforehand but that now decides to invest along with a randomly drawn measure \( \delta \) of other sellers. That is, after this coordinated investment decision, the fraction of investing sellers increases from \( \mu \) to \( \mu + \delta \). Then, the seller’s incentive to innovate is computed as

\[
\Delta^m = \left[ \frac{1}{2} \pi' + (\mu + \delta) u' + (1 - (\mu + \delta)) u - (C_s + \tau_s) \right] \\
- \left[ \frac{1}{2} \pi + \mu u' + (1 - \mu) u - (C_s + \tau_s) \right] \\
= \frac{1}{2} (\pi' - \pi) + \delta (u' - u) = \Delta^n + \delta (u' - u).
\]

Hence, as in the previous subsection, intermediated trade gives higher incentives to innovate if investment improves the buyers’ net gain from trade, i.e., if \( u' > u \). Note, however, that if a seller modifies unilaterally his investment decision, there is no difference between intermediated and non-intermediated trade: for \( \delta \to 0 \), \( \Delta^m = \Delta^n \). The intuition is simple: an individual seller is too small to have its investment decision driving platforms to change their optimal fee. We summarize our preceding analysis by the following remark:

**Remark 2** If platforms are not able to condition their fees on the sellers’ investment decisions, the market microstructure affects incentives to innovate only if investments result from
a coordinated decision among sellers. What we add here is that it is not necessary to assume that coordination takes place among all sellers: it is sufficient to have coordination within a subgroup of sellers (drawn independently from the total population of sellers); the larger the size of this subgroup, the larger the difference between incentives to innovate in intermediated and non-intermediated trade.

3.3.2 Differential sellers’ membership fee

We now show that if platforms are able to price discriminate, the market microstructure has an impact on individual incentives to invest. We assume here that platforms can set $M_s' \neq M_s$. Under this assumption, we solve again the system of equations (9) to (12) to find the equilibrium values of $n_b^i$ and $n_s^i$, expressed as functions of the membership fees set by the two platforms. Platform $i$ now chooses $M_b^i, M_s'^i, \text{ and } M_s^i$ to maximize

$$\Pi_i = n_b^i (M_b^i - C_b) + n_s^i [\mu M_s'^i + (1 - \mu) M_s^i - C_s].$$

The symmetric solution to the system made of the six first-order conditions gives the following simple equilibrium fees:

$$M_b^* = C_b + \tau_b - \mu \pi' - (1 - \mu) \pi,$$

$$M_s'^* = C_s + \tau_s - u' \text{ and } M_s^* = C_s + \tau_s - u.$$ 

As before, $n_b^1 = n_b^2 = n_s^1 = n_s^2 = \frac{1}{2}$. We can then compute the equilibrium buyer and seller surplus (gross of opportunity and potential investment costs):

$$v_s^* = \begin{cases} \frac{1}{2} \pi' + u' - (C_s + \tau_s) & \text{if the seller has invested,} \\ \frac{1}{2} \pi + u - (C_s + \tau_s) & \text{if the seller has not invested,} \end{cases}$$

$$v_b^* = \mu \left(\frac{1}{2} u' + \pi'\right) + (1 - \mu) \left(\frac{1}{2} u + \pi\right) - (C_b + \tau_b).$$

We notice here that, because of price discrimination, the seller surplus does not depend on the proportion of investing sellers ($\mu$). We can thus compute the incentive to innovate as the difference between the surplus of an investing seller and of a non-investing seller, i.e.,

$$\Delta^{in} = \left[\frac{1}{2} \pi' + u' - (C_s + \tau_s)\right] - \left[\frac{1}{2} \pi + u - (C_s + \tau_s)\right]$$

$$= \frac{1}{2} \left(\pi' - \pi\right) + (u' - u).$$

If we had free platforms instead of for-profit platforms, an investing seller and a non-investing seller would respectively earn a surplus equal to $\frac{1}{2} \pi'$ and to $\frac{1}{2} \pi$. It follows that the
incentive to innovate when trade is not intermediated is equal to $\Delta^n = \frac{1}{2} (\pi' - \pi)$. Comparing incentives to innovate, we find that $\Delta^m = \Delta^n + (u' - u)$. We come thus to the same conclusion as in Proposition 1: intermediated trade gives higher incentives to innovate if investment improves the buyers’ net gain from trade, i.e., if $u' > u$. The following remark states the main insight of this subsection:

**Remark 3** If platforms price-discriminate between sellers of different investment levels, each seller internalizes the change in buyer surplus resulting from its investment even if sellers do not coordinate their investment decisions.

### 4 Competitive bottlenecks when sellers multihome

In this section, we analyze investment incentives in market environments in which sellers have the possibility to multihome. As noted by Evans (2003), personal computers constitute a typical example of this situation: end-users (i.e., buyers) singlehome (they almost always use a single operating system), while application developers (i.e., sellers) do multihome.\(^\dagger\) Other examples include retail chains that can locate in competing shopping malls, firms that list in competing yellow pages, and shops that accept competing credit cards (provided that consumers hold only one card).

#### 4.1 Equilibrium for given investment levels

At stage 2, buyers are in the same situation as in the previous section: as they singlehome, they divide into two groups, one that goes to platform 1 (i.e., buyers located between 0 and $b_{12}$) and the other that goes to platform 2 (i.e., buyers located between $b_{12}$ and 1). On the other hand, because sellers now have the possibility to multihome, they are divided into three groups: those located between 0 and $s_{20}$ visit platform 1 only, those located between $s_{20}$ and $s_{10}$ visit both platforms, and those located between $s_{10}$ and 1 visit platform 2 only. Recalling the identification of the various indifferent agents, we derive the number of buyers and sellers visiting each platform respectively as

\[
n_{b}^i = \frac{1}{2} + \frac{v_i^b - v_j^b}{2\tau_b} \quad \text{and} \quad n_{s}^i = \frac{v_i^s}{\tau_s},
\]

\(^\dagger\)See Lerner (2002) for data about the number of developers that develop for various operating systems.
or equivalently, using the expressions for buyer and seller surplus (1) and (2), as

\[ n_b^i = \frac{1}{2} + \frac{u(n_s^i - n_s^j) - (M_b^i - M_b^j)}{2\tau_b} \quad \text{and} \quad n_s^i = \frac{n_b^i \pi - M_b^i}{\tau_s}. \]

Solving this system of four equations in four unknowns, we get

\[ n_b^i = \frac{1}{2} + \frac{u(M_s^j - M_s^i) + \tau_s(M_b^i - M_b^j)}{2(\tau_b \tau_s - u \pi)} \quad \text{and} \quad n_s^i = \frac{\pi}{\tau_s} \left( \frac{1}{2} + \frac{u(M_s^j - M_s^i) + \tau_s(M_b^i - M_b^j)}{2(\tau_b \tau_s - u \pi)} \right) - \frac{M_b^i}{\tau_s}. \]

Comparing expressions (5) and (13) and supposing that membership fees do not change, we observe that the number of buyers is the same whether sellers are allowed to multihome or not. As for sellers, the comparison of expressions (6) and (14) reveals that (still supposing that membership fees do not change) the number of sellers at platform \( i \) is larger when sellers are allowed to multihome if \( \pi > M_s^i + M_s^j + \tau_s \).

As in the previous section, our reference point is the situation of two open platforms located at the extremes of the unit interval (that is, they have the same locations as for-profit platforms). Suppose access is free. Setting \( M_b^i = M_b^j = M_s^i = M_s^j = 0 \) in expressions (13) and (14), we compute the numbers of buyers and sellers on each platform as

\[ n_b^1 = n_b^2 = \frac{1}{2} \quad \text{and} \quad n_s^1 = n_s^2 = \frac{1}{2\tau_s} \pi. \]

It follows that the (per platform) net surplus of sellers and buyers are equal to

\[ v_s^n = \frac{1}{2} \pi \quad \text{and} \quad v_b^n = \frac{1}{2\tau_s} \pi u. \]

Assuming that \( \pi < 2\tau_s \), we have that the sellers located between \( 1 - \pi / (2\tau_s) \) and \( \pi / (2\tau_s) \) visit both open platforms, whereas the other sellers visit only the platform close to their location. As long as the equilibrium fee set by for-profit platforms, \( M_s^i \), is positive, open platforms attract more sellers than for-profit platforms (which also means that more sellers multihome if trade is organized through open platforms).

Let us now turn to the pricing stage of the game in which platforms are for profit. Each platform \( i \) solves the problem \( \max_{M_b^i, M_s^i} \Pi^i \) where

\[ \Pi^i = (M_b^i - C_b) n_b^i(M_b^i, M_b^j, M_s^i, M_s^j) + (M_s^i - C_s) n_s^i(M_b^i, M_b^j, M_s^i, M_s^j). \]
Equilibrium prices are\textsuperscript{18}

\[M_b^* ≡ M_b^{1*} = M_b^{2*} = C_b + \tau_b - \frac{\tau}{4\tau_s} (3u + \pi - 2C_s),\]
\[M_s^* ≡ M_s^{1*} = M_s^{2*} = \frac{1}{2} C_s + \frac{1}{4}(\pi - u).\]

On the seller side, platforms have monopoly power. If the intermediary focused only on sellers, he would charge a monopoly price equal to \(C_s/2 + \pi/4\) (assuming that each seller would have access to half of the buyers and, therefore, would have a gross willingness to pay equal to \(\pi/2\)). We observe that this price is adjusted downward by \(u/4\) when the indirect network effect that sellers exert on the buyer side is taken into account (and remains positive as long as \(\pi + 2C_s > u\)). Sellers are subsidized if \(u > \pi\), i.e. the indirect network effect is stronger on the buyer than on the seller side. Similarly, on the buyer side, platforms charge the Hotelling price, \(C_b + \tau_b\), less a term that depends on the size of the indirect network effects.

It is useful to compare price changes in the competitive bottleneck model to those in the two-sided singlehoming model. In equilibrium, we observe that the membership fee for sellers is increasing in the strength of the indirect network effect \((\partial M_s^* / \partial \pi > 0)\), whereas it is constant in the two-sided singlehoming model. This is due to the monopoly pricing feature on the multihoming side. Everything else equal, if sellers are multihoming, the platform operators directly appropriate part of the rent generated on the multihoming side by setting higher membership fees. This is not the case in the singlehoming world, where the membership fee does not react to the strength of the network effect on the same side since platforms compete for sellers (and buyers). This observation is relevant for the analysis of investment incentives below.

It follows that at equilibrium,

\[n_b^{1*} = n_b^{2*} = \frac{1}{2},\]
\[n_s^{1*} = n_s^{2*} = \frac{1}{4\tau_s} (u + \pi - 2C_s).\]

\textsuperscript{18}Firm’s best responses are implicitly defined by the first-order conditions which can be expressed as \(M_b^s = [- (u + \pi) M_b^s + uM_b^s + \tau_s M_b^s - \pi (u - C_s) + \tau_s (\tau_b + C_b)]/[2\tau_s],\) \(M_s^1 = [- (u + \pi) \tau_s M_s^1 + u\pi M_s^1 + \pi \tau_s M_s^2 - \pi u (\pi + C_s) + u\tau_s C_b + (\pi + 2C_s) \tau_b \tau_s]/[2 (2\tau_s \tau_s - w\pi)].\) Second-order conditions require that \(8\tau_s \tau_s > \pi^2 + u^2 + 6\pi u.\) This condition is also sufficient to have a unique and stable interior equilibrium at stage 2. Note that in the special case where parameters are symmetric on both sides of the market, i.e. \(\tau_s = \tau_b \equiv \tau\) and \(u = \pi,\) this inequality simplifies to \(\tau > u = \pi.\)
Thus we must have \( 0 < n^i_s < 1 \Leftrightarrow 2C_s < u + \pi < 2C_s + 4\tau_s \) for obtaining an interior solution.\(^\text{19}\) Under these conditions, the equilibrium net surplus of sellers and buyers (gross of transportation cost and for one platform) are equal to:

\[
\begin{align*}
    v^*_s &= \frac{1}{4}(u + \pi) - \frac{1}{2}C_s, \\
    v^*_b &= \frac{1}{4\tau_s}(u^2 + 4\pi u + \pi^2 - 2(u + \pi)C_s) - \tau_b - C_b.
\end{align*}
\]

Note that \( v^*_s \) is the per platform seller’s surplus. As conditions (3) and (4) imply that \( n^i_s > 1/2 \), the sellers located between \( 1 - (v^*_s/\tau_s) \) and \( v^*_s/\tau_s \) multihome and, therefore, earn a surplus of \( 2v^*_s \). On the other hand, \( v^*_s \) is the surplus earned by the sellers located between 0 and \( 1 - (v^*_s/\tau_s) \), who choose to visit platform 1 only, and by the sellers located between \( v^*_s/\tau_s \) and 1, who choose to visit platform 2 only. We observe that \( v^*_s \) and \( v^*_b \) are increasing in the net gain of the other side and in the net gain of the own side. The intermediaries’ equilibrium profits are

\[
\Pi^* = \frac{1}{16\tau_s}(8\tau_b\tau_s - (\pi^2 + u^2 + 6\pi u) + 4C_s^2) > 0.
\]

### 4.2 Seller incentives to innovate

As before, we compare the sellers’ incentives to innovate under two different organizations of the trading platforms, a situation in which there are two for-profit platforms and a situation in which there are two free open platforms. In line with our previous analysis, we assume for simplicity that all sellers agree upon the same investment level. However, we can again show (see Appendix 3) that our results carry over if we relax the assumption of coordinated investment decisions.

Using the same notation as in the previous section, with two strategic intermediaries, sellers are willing to pay up to the increase in their (equilibrium) net surpluses \( \Delta^m = v_s(y') - v_s(y) \). Here, we need to distinguish between the sellers who multihome and those who singlehome at equilibrium. The increase in net surplus is given by

\[
\Delta^m = \begin{cases} 
    \frac{1}{2} [u(y') - u(y) + \pi(y') - \pi(y)] & \text{for multihoming sellers,} \\
    \frac{3}{4} [u(y') - u(y) + \pi(y') - \pi(y)] & \text{for singlehoming sellers.}
\end{cases}
\]

\(^{19}\)More precisely, conditions (3) and (4) rewrite here as: \( K/(8\tau_s) < \tau_b < K/(6\tau_s) \) with \( K \equiv u^2 + 4\pi u + \pi^2 - 2(u + \pi)C_s - 4\tau_s C_b \) and \( \frac{1}{4}(u + \pi - 2C_s) < \tau_s < \frac{1}{2}(u + \pi - 2C_s) \).
If trade takes place on two open platforms, the increase in the sellers’ net surpluses is
\[
\Delta^n = \begin{cases} 
\pi(y') - \pi(y) & \text{for multihoming sellers,} \\
\frac{1}{2} (\pi(y') - \pi(y)) & \text{for singlehoming sellers.}
\end{cases}
\]

Comparing the expressions for \(\Delta^m\) and \(\Delta^n\), we obtain that (whether sellers multihome or singlehome under both organizations) a seller’s willingness to invest is larger (provided all other sellers invest the same) if trade occurs via for-profit platform rather than via open platforms if the investment leads to a larger increase of the buyers’ net surplus than of the sellers’ net surplus. We indicated above that platform size and thus the number of multi- and singlehoming sellers depend on the type of platform organization. In particular, more sellers multihome if platforms are open. However, if in both types of platform organization sellers who singlehome are decisive to determine the individual contribution to investment (which is assumed to be the same for each seller), then we obtain the following result with respect to sellers’ incentives.\textsuperscript{20}

**Proposition 2** Suppose that sellers who singlehome determine investment levels. In the competitive bottleneck model in which sellers are on the multihoming side, for profit trading platforms give stronger incentives for sellers to innovate if and only if the change of the buyers’ surplus is larger than the change of the sellers’ surplus, i.e.
\[
\Delta^m > \Delta^n \iff u(y') - u(y) > \pi(y') - \pi(y).
\] (18)

Note that this condition is more demanding than the corresponding rule in the single-homing environment, rule (8), provided that profits are increasing in the investment level \(y\). This is due to the fact that the sellers’ surplus can be better extracted by for-profit platforms when the sellers can multihome but that they are less critical for the overall success of the platform. The intuition is that each platform does not directly react to a change in membership fee charged by the competitor on the multihoming side (i.e. for given \(n_i\)) so that there are no direct multiplier effects between \(M_1^s\) and \(M_2^s\). As an equilibrium outcome in our linear model, a larger \(\pi\) does not affect sellers’ membership fees if sellers singlehome but leads to higher membership fees if they multihome. This implies that an increase in investment leads to an increase of the equilibrium membership fee on the seller side, which makes the above inequality more demanding than the corresponding inequality under two-sided singlehoming.

\textsuperscript{20}The sellers located in \([1 - \pi/(2\tau_s), 1 - u_i^*/\tau_s]\) and in \([\pi/(2\tau_s), u_i^*/\tau_s]\) multihome if platforms are open, but singlehome if platforms are strategic. For them, the condition for incentives to innovate to be higher under intermediated trade is more stringent: \(\Delta^m > \Delta^n \iff u(y') - u(y) > 3[\pi(y') - \pi(y)]\).
5 Competitive bottlenecks when buyers multihome

We analyze the same model as in the previous section with the only difference that the role of buyers and sellers is reversed, that is, sellers singlehome and buyers multihome. For instance, every Sunday morning, there are two flea markets in Brussels; their locations are sufficiently close for consumers to be able to visit both on the same morning; however, sellers are not mobile and stay put on a single market. Similarly, owner-managed shops may set up in only one of the shopping areas but consumers may be able to make it to both areas for their shopping. Such a situation also arises in cases in which sellers sign exclusivity contracts with platforms but where buyers multihome. Hence, for given investment levels only the buyer and seller indices (as well as \( u \) and \( \pi \)) have to be reversed and the analysis of the previous section applies. Rewriting expression (17), we have that the equilibrium net surplus of sellers (gross of transportation cost) is now equal to:

\[
v^*_s = \frac{1}{4\pi_b}(u^2 + 4\pi u + \pi^2 - 2(u + \pi)C_b) - \tau_s - C_s.
\]

On the other hand, if trade takes place via two open platforms, the surplus of sellers is obtained from expression (15):

\[
v^n_s = \frac{1}{2\pi_b}\pi u.
\]

Again we compare the sellers’ incentives to innovate under the two organizations of the trading platforms. If buyers are on the multihoming side, we can show that *trade via for-profit platforms lead to stronger investment incentives than via open platforms if the joint surplus of buyers and sellers increases*, which has to be the case under any potentially welfare-improving increase of the investment level.

**Proposition 3** *In the competitive bottleneck model in which sellers are on the singlehoming side, for-profit trading platforms give stronger incentives for sellers to innovate if and only if the joint buyers’ and sellers’ surplus increases, i.e.*

\[
\Delta^m > \Delta^n \iff u(y') - u(y) + \pi(y') - \pi(y) > 0.
\]  

**Proof.** Incentives to innovate are computed as: \( \Delta^m = v^*_s(y') - v^*_s(y) \) and \( \Delta^n = v^n_s(y') - v^n_s(y) \). Using the above expressions for \( v^*_s \) and \( v^n_s \), developing and simplifying, we find

\[
\Delta^m - \Delta^n = \frac{1}{4\pi_b}(u(y') - u(y) + \pi(y') - \pi(y))(u(y') + u(y) + \pi(y') + \pi(y) - 2C_b) .
\]
As the second bracketed term is positive (otherwise, for-profit platforms would not be able to make a profit), we check condition (19).

Note that this condition is less demanding than rule (8). Here, the membership fee on the singlehoming side is substantially lower since platforms compete more fiercely for the singlehoming side. However, the reaction of the membership fee to the level of investment is relevant for investment incentive. Since sellers are critical for the success of a platform, intermediaries are more likely to refrain from increasing the membership fee.

6 Applications

In line with most of the literature on two-sided markets, we did not provide so far a microfoundation of $u$ and $\pi$. To fill the gap, we use a simple model according to which each seller offers an independent product, i.e. a product that is neither a substitute nor a complement for the other products. Each consumer has independent variable demand for each of the products. Suppose that inverse demand for each product is given by $P(q) = \max\{1 - q^\alpha, 0\}$ for $\alpha > 0$. If $\alpha = 1$, demand is linear; if $0 < \alpha < 1$, demand is convex, and if $\alpha > 1$, demand is concave where positive. Suppose that marginal cost of production is constant and equal to $0 \leq c < 1$.

We consider four specific types of investments: sellers can invest in order to (1) reduce their marginal cost of production, (2) improve the quality of their product, (3) enhance their ability to price discriminate, or (4) expand demand. For each type of investment, we examine whether incentives to invest are higher under intermediated or non-intermediated trade. At the end of the section, we collect our findings with respect to these investment examples.

To facilitate the analysis, it is useful to summarize the results of Propositions 1 to 3: incentives to innovate are stronger under intermediated trade if and only if

- $u(y') - u(y) > 0$ in a market in which both sides singlehome,
- $u(y') - u(y) > \pi(y') - \pi(y)$ in a market in which buyers singlehome and sellers can multihome,
- $u(y') - u(y) + \pi(y') - \pi(y) > 0$ in a market in which sellers singlehome and buyers can multihome.

The most simple specification of demand is the linear case. In this case, monopoly pricing implies that $u(y) = \pi(y)/2$ for all $y$ which may affect costs or demand. Hence, in a market
in which both sides singlehome or in which sellers singlehome and buyers can multihome, investment incentives are stronger under intermediated trade if and only if the investment increases the gross profit per buyer, \( \pi(y') - \pi(y) > 0 \). The reverse holds in a market in which buyers singlehome and sellers can multihome. Below, we look at a more flexible specification of demand.

### 6.1 Cost reducing R&D

In this example, the investment level determines \( c \). Each sellers’ decision problem at the last stage reduces to the simple monopoly maximization problem \( \max_q \pi = (1 - c - q^\alpha) q \). The profit-maximizing quantity is computed as:

\[
q = \left( \frac{1 - c}{\alpha + 1} \right)^{\frac{1}{\alpha}}.
\]

We obtain the firm’s profit and the consumer surplus at the profit-maximizing quantity as a function of \( c \):

\[
\pi(c) = \frac{\alpha (1 - c)}{\alpha + 1} \left( \frac{1 - c}{\alpha + 1} \right)^{\frac{1}{\alpha}},
\]

\[
u(c) = \frac{\alpha (1 - c)}{(\alpha + 1)^2} \left( \frac{1 - c}{\alpha + 1} \right)^{\frac{1}{\alpha}} = \frac{1}{\alpha + 1} \pi(c).
\]

Suppose there exists a process innovation that allows sellers to decrease the marginal cost of production from \( c \) to \( c' \), with \( 0 < c' \leq c \). Both the firm’s profit and the consumer surplus increase as a result of the cost reduction; that is, \( u(c') - u(c) > 0 \) and \( \pi(c') - \pi(c) > 0 \). It follows that intermediated trade provides higher incentives to invest in cost-reduction when both sides singlehome (from Proposition 1) and when sellers singlehome while buyers can multihome (from Proposition 3). As for the situation where buyers singlehome and sellers can multihome, incentives are higher under intermediated trade if and only if (from Proposition 2):

\[
\frac{1}{\alpha + 1} (\pi(c') - \pi(c)) > \pi(c') - \pi(c),
\]

which is never true as, by assumption, \( \alpha > 0 \). It follows that in the case where buyers singlehome and sellers can multihome, incentives to invest in cost-reduction are lower under intermediated trade.
6.2 Quality improving R&D

By simply relabeling variables, we can replicate the analysis for a type of quality improving R&D. Suppose that there exists a product innovation that shifts the inverse demand curve outward; namely, consider quality \( s \geq 0 \) with \( P(q) = 1 + s - q^\alpha \). Marginal costs are here set equal to zero. Then, the profit maximization problem of each monopoly seller becomes: \( \max_q \pi = (1 + s - q^\alpha) q \), which is made equivalent to the above analysis by simply substituting \(-c\) for \( s\). The results of the previous subsection therefore carry over.

6.3 Investment in price discrimination

The third type of investment we consider consists in joint data collection activities and in information sharing agreements among sellers. This investment allows sellers to practice some form of price discrimination and capture thereby a share \( 0 \leq \beta \leq 1 \) of the consumer’s surplus at a given price \( p \). We can think of each seller setting a two-part tariff of the form: \( T(p,q) = \beta CS(p) + pq \), where \( CS(p) \) is the consumer’s surplus at price \( p \). A few lines of computation establish that

\[
CS(p) = \frac{\alpha}{\alpha + 1} \left( 1 - p \right)^{\frac{\alpha + 1}{\alpha}}.
\]

The innovation allows sellers to capture a larger share of the consumer’s surplus (i.e., to increase \( \beta \)). For a given value of \( \beta \), the seller chooses \( p \) so as to maximize (to ease the computations, we set the marginal cost to zero):

\[
\max_p \pi(p, \beta) = p \left( 1 - p \right)^{\frac{1}{\alpha}} + \frac{\alpha}{\alpha + 1} \left( 1 - p \right)^{\frac{\alpha + 1}{\alpha}}.
\]

The profit-maximizing price is easily found as

\[
p^* (\beta) = \frac{\alpha (1 - \beta)}{1 + \alpha (1 - \beta)}.
\]

We compute the net gain from trade for each seller and for each buyer respectively as:

\[
\pi(\beta) = \pi(p^* (\beta), \beta) = \beta CS(p^* (\beta)) + p^* (\beta) \left( 1 - p^* (\beta) \right)^{\frac{1}{\alpha}}
\]

\[
= \frac{\alpha}{\alpha + 1} \left( 1 + \frac{\alpha}{\alpha + 1} \right)^{\frac{1}{\alpha}},
\]

\[
u(\beta) = (1 - \beta) CS(p^* (\beta)) = (1 - \beta) \frac{\alpha}{\alpha + 1} \left( 1 + \frac{1}{\alpha + 1 \left( 1 - \beta \right) - 1} \right)^{\frac{\alpha + 1}{\alpha}}.
\]

\footnote{We check that \( p^* (0) = \alpha/(1 + \alpha) \) (profit-maximizing uniform price) and \( p^* (1) = 0 \) (the variable part of the tariff is equal to the marginal cost in case of perfect price discrimination).}
Observe that $\pi(\beta)$ increases and $u(\beta)$ decreases with $\beta$:

$$\frac{d}{d\beta}u(\beta) = -\frac{\beta\alpha}{\alpha + 1} \left( \frac{1}{1 + \alpha (1 - \beta)} \right)^{2\alpha+1} < 0.$$

Therefore, $u(\beta') - u(\beta) < 0 < \pi(\beta') - \pi(\beta)$, which implies, using Propositions 1 and 2, that intermediated trade provides lower incentives to innovate in price discrimination when both sides singlehome and when buyers singlehome while sellers can multihome. As for the third case (sellers singlehome, buyers can multihome), we observe that total surplus increases with $\beta$:

$$\pi(\beta) + u(\beta) = \frac{\alpha}{\alpha + 1} \left( \frac{1}{1 + \alpha (1 - \beta)} \right)^{1\alpha} \left( \frac{2 - \beta + \alpha (1 - \beta)}{1 + \alpha (1 - \beta)} \right),$$

$$\frac{d}{d\beta} (\pi(\beta) + u(\beta)) = \frac{\alpha (1 - \beta)}{(1 + \alpha (1 - \beta))^2} \left( \frac{1}{1 + \alpha - \alpha \beta} \right)^{\frac{1}{\alpha}} > 0.$$  

Applying Proposition 3, we thus have that here intermediated trade provides higher incentives to innovate.

### 6.4 Investment in demand expansion

In this simple model, we model demand expansion as an increase in $\alpha$. Note that the parameter $\alpha$ determines also the curvature of the demand function given by $q = (1 - p)^{1/\alpha}$. Here, a higher $\alpha$ not only increases the average willingness-to-pay per unit but, as we will see, also affects the rent distribution between consumers and sellers in the profit-maximizing solution. We obtain that

$$\frac{dq}{d\alpha} = -\frac{1}{\alpha^2} (1 - p)^{\frac{1}{\alpha}} \ln (1 - p) > 0,$$

which means that at a given price $p$, the quantity demanded increases as $\alpha$ increases.

Suppose that the sellers’ investment has the effect of increasing $\alpha$ from $\alpha_0$ to $\alpha_1 = \alpha_0 + \Delta \alpha$. If $\Delta \alpha$ is small, the impact of the investment on the firm’s profit and the consumer surplus can be approximated by $\pi' (\alpha) \Delta \alpha$ and $u' (\alpha) \Delta \alpha$ respectively. To simplify the exposition, let
us fix again $c = 0$. We compute:

$$
\pi'(\alpha) = \frac{1}{\alpha(\alpha+1)^{2/3}} \left( \frac{1}{\alpha+1} \right)^{2/3} \ln (\alpha + 1) > 0,
$$

$$
u'(\alpha) = \frac{1}{\alpha(\alpha+1)^{3/3}} \left( \frac{1}{\alpha+1} \right)^{1/3} \frac{1}{\alpha} [(\alpha + 1) \ln (\alpha + 1) - \alpha^2] > 0 \text{ if and only if } \alpha < 1.537
$$

$$
\pi'(\alpha) - u'(\alpha) = \frac{1}{\alpha(\alpha+1)^{4/3}} \left( \frac{1}{\alpha+1} \right)^{4/3} \frac{1}{\alpha} [(\alpha + 1) \ln (\alpha + 1) + \alpha] > 0,
$$

$$
\pi'(\alpha) + u'(\alpha) = \frac{1}{\alpha(\alpha+1)^{4/3}} \left( \frac{1}{\alpha+1} \right)^{4/3} \frac{1}{\alpha} [(\alpha + 2) (\alpha + 1) \ln (\alpha + 1) - \alpha^2] > 0.
$$

We observe that the consumer surplus has a U-inverted shape with respect to $\alpha$. Hence, for small values of $\alpha$ (namely, for $\alpha < 1.537$), the consumer surplus increases in $\alpha$, meaning that incentives to innovate are higher under intermediated trade when both sides singlehome; the opposite result prevails for larger values of $\alpha$ (namely, for $\alpha > 1.537$). Hence, at a large $\alpha_0$, buyers obtain consumer surplus $u(\alpha_0)$ with $u'(\alpha_0) < 0$. Then a larger $\alpha_1$ reduces $u$ and makes platforms increase the sellers membership fees $M^*_i$ in response.

We also observe that $\pi'(\alpha) - u'(\alpha) > 0$ and $\pi'(\alpha) + u'(\alpha) > 0$ for all values of $\alpha$. The former inequality implies that incentives to innovate are lower under intermediated trade when sellers can multihome; the latter implies that incentives to innovate are higher under intermediated trade when buyers can multihome.

### 6.5 Summary

We collect the results of the four applications in the following remark.

**Remark 4** Whether investment incentives are stronger under intermediated or non-intermediated trade depends on which side of the market singlehomes and on the nature of the investment. In our parametric specification, results are summarized by the following table (where “+” stands for stronger seller investment incentives if platforms are for-profit).

<table>
<thead>
<tr>
<th></th>
<th>cost reduction</th>
<th>quality improvement</th>
<th>price discrimination</th>
<th>demand expansion</th>
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</thead>
<tbody>
<tr>
<td>sellers multihome</td>
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<tr>
<td>buyers singlehome</td>
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<tr>
<td>both sides singlehome</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+/–*</td>
</tr>
<tr>
<td>sellers singlehome</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>buyers multihome</td>
<td></td>
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</tbody>
</table>

* “+” if demand sufficiently convex (“–” otherwise)
7 Conclusion

In this paper, we have analyzed whether and how the fact that products are not sold on open platforms but on competing for-profit platforms affects sellers’ investment incentives. Investments in cost reduction, quality, or marketing measures are here the joint and coordinated efforts by sellers (in the form of horizontal arrangements). In particular, they may take the form of R&D cooperatives. We show that, in general, the trading environment is not neutral to such investment incentives. Alternatively, if platforms can discriminate between investing and non-investing sellers, investment decisions may be non-coordinated (as formally analyzed under two-sided singlehoming and under competitive bottlenecks when sellers can multihome).

We build a model with many manufacturers and consumers and two competing intermediaries who charge membership or access fees on both sides of the market. We compare this situation to an environment in which manufacturers and consumers have free access to platforms. Clearly, the presence of for-profit intermediaries reduces the rents that are available in the market. Therefore, one might suspect that sellers have weaker investment incentives with competing for-profit platforms. However, this is not necessarily the case. The reason is that investment incentives affect the size of the network effects and thus competition between intermediaries.

In particular, we show that the relative strength of investment incentives depends on which side of the market singlehomes and on the nature of the R&D cooperation. For instance, if both sides singlehome, incentives to invest in cost reduction are stronger with competing for-profit platforms, whereas incentives to invest in consumer targeting (that, e.g., improve the possibility of price discrimination) are weaker. Our results are relevant for the debate on innovation policy. For instance, a large part of the discussion around the protection of intellectual property focuses exclusively on seller surplus. However, as we have shown, innovators may also benefit from higher consumer surplus as some of its improvements are internalized by sellers in the form of more favorable conditions of access to the platform.

Our formal analysis suffers from a number of limitations that future research should endeavor to address. First, our analysis only allows for membership or participation fees and platforms do not charge for usage. In many real-world examples, platform also charge for usage on at least one side of the market (e.g., video game platforms receive royalties from game developers for each game sold). Unfortunately, a meaningful analysis of platforms which
charge for both usage and membership is involved and we have preferred, in this paper, to concentrate on a simple framework.\textsuperscript{22}

Second, we have assumed that whether one side of the market single- or multihomes is exogenously given. In some markets, this may be technologically given. However, in other markets, some buyers and sellers singlehome whereas others multihome. As Section 2 has clarified, we have looked at three extreme situations. One may want to generalize our analysis to cases where only a share of buyers or of sellers have the possibility to multihome. In addition, single- versus multihoming may be endogenously determined. Future work may want to look at this issue.\textsuperscript{23}

Finally, the scope of our analysis is limited by our assumption that sellers take their pricing decisions independently of one another. In Appendix 1, we show that our results carry over to a situation of imperfect competition among sellers captured by a negative direct network effect among sellers. This can be interpreted as a congestion effect on the platform. In our specification, sellers’ profits decrease linearly with the number of sellers present on a platform, but, as we show, pricing decisions remain independent. A more general approach would be to consider strategic interaction in pricing decisions (e.g., that sellers produce imperfectly differentiated products). However, such interaction makes the platform choice game much more complex to solve than in the present setting. We leave this for future research.

While the precise comparison of investment incentives under the two alternative governance structures (for-profit versus open) of platforms is partly due to our linear specification, the general insight of the paper is that profit-maximizing intermediaries adjust their access fees strategically to seller investments with the effect that for-profit intermediation may provide stronger seller incentives to invest. This insight is not restricted to a set-up with competing intermediaries and seller coordination at the investment stage. With respect to the number of intermediaries, we note that already in the monopoly model the intermediary adjusts prices to the strength of indirect network effects. Further work may want to focus on the role of competition between intermediaries for seller investment incentives.

\textsuperscript{22}For a discussion of the use of different price instruments, see Rochet and Tirole (2006). Also timing issues are likely to affect equilibrium prices.

\textsuperscript{23}Armstrong and Wright (2007) provide such an analysis with exogenous investment levels.
Appendix

Appendix 1: Competition among sellers

We introduce competition on the sellers’ side in the following simple way: we now express the seller’s surplus on platform \( i \) as

\[
v_s^i = n_b^i \pi - \gamma n_s^i - M_s^i,
\]

with \( \gamma > 0 \). According to this formulation, each seller still earns \( \pi \) per buyer he interacts with, but now loses \( \gamma \) per seller present on his platform. We can think of some form of congestion on the platform to explain the latter effect. It follows that, all other things equal, a seller prefers the platform visited by the lowest number of sellers. In that sense, sellers are competing with one another for platform access. To keep the linear structure, we model “competition” as a linear congestion externality. For-profit platforms take this negative direct network effect into account by raising the sellers’ subscription fee.

We show here that the results of Propositions 1 to 3 remain valid under this alternative specification.

Two-sided singlehoming. The expressions for the numbers of buyers and sellers at the two platforms are now given by:

\[
n_b^i = \frac{1}{2} + \frac{(2n_s^i - 1)u - (M_b^i - M_b^j)}{2\tau_b},
\]

\[
n_s^i = \frac{1}{2} + \frac{(2n_b^i - 1)\pi - (M_s^i - M_s^j)}{2(\tau_s + \gamma)}.
\]

Solving this system of linear equations we obtain

\[
n_b^i = \frac{1}{2} + \frac{u(M_b^i - M_b^j) + (\gamma + \tau_s)(M_s^i - M_s^j)}{2(\tau_b(\gamma + \tau_s) - u\pi)},
\]

\[
n_s^i = \frac{1}{2} + \frac{\pi(M_b^i - M_b^j) + \tau_b(M_s^i - M_s^j)}{2(\tau_b(\gamma + \tau_s) - u\pi)}.
\]

Comparing with expressions (5) and (6) obtained in the paper (with \( \gamma = 0 \)), we observe that the only difference is that \( (\gamma + \tau_s) \) is substituted for \( \tau_s \). Therefore, we obtain equilibrium membership fees from the previous ones by just making this substitution; that is:

\[
M_s^* = C_s + \gamma + \tau_s - u \quad \text{and} \quad M_b^* = C_b + \tau_b - \pi.
\]
It follows that at equilibrium, \( n_1^* = n_2^* = \frac{1}{2} \) and \( n_1^s = n_2^s = \frac{1}{2} \), so that the equilibrium net surplus of sellers and buyers (gross of transportation cost) are equal to:

\[
\begin{align*}
\nu_s^* &= \frac{1}{2} \pi + u - (C_s + \tau_s) - \frac{\gamma}{2}, \\
\nu_b^* &= \frac{1}{2} \pi + u - (C_b + \tau_b).
\end{align*}
\]

If interaction takes place on open platforms, then it easily found (by setting \( M_s^i = M_s^j = M_b^i = M_b^j = 0 \) in the above expressions) that

\[
\begin{align*}
\nu_s^i &= \frac{1}{2} (\pi - \gamma) \quad \text{and} \quad \nu_b^i = \frac{1}{2} u,
\end{align*}
\]

Considering now the sellers’ incentives to innovate, we observe that nothing changes with respect to the previous case as the competition effect (which is assumed to remain unaffected by the investment level) comes as an additive term in the sellers’ surplus functions. So, when computing incentives to innovate \( \Delta_n \) and \( \Delta_m \), the term in \( \gamma \) disappears and we are left with the same expressions as before, implying that the result of Proposition 1 carries over.

**Competitive bottlenecks when sellers multihome.** It is easily checked that the equilibrium numbers of buyers and sellers on each platform can be derived from expressions (13) and (14) by simply substituting \( (\gamma + \tau_s) \) for \( \tau_s \):

\[
\begin{align*}
\bar{n}_b^i &= \frac{1}{2} + \frac{u(M_s^i - M_b^i) + (\gamma + \tau_s)(M_b^j - M_b^i)}{2(\tau_b (\gamma + \tau_s) - u \pi)}, \\
\bar{n}_s^i &= \frac{\pi (\gamma + \tau_s)}{(\gamma + \tau_s)} \left( \frac{1}{2} + \frac{u(M_s^i - M_b^i) + (\gamma + \tau_s)(M_b^j - M_b^i)}{2 (\tau_b (\gamma + \tau_s) - u \pi)} \right) - \frac{M_s^i}{(\gamma + \tau_s)}.
\end{align*}
\]

Proceeding as in Section 4, we obtain

\[
\begin{align*}
M_b^{1*} &= M_b^{2*} = \tau_b + C_b - \frac{\pi}{4(\gamma + \tau_s)} (3u + \pi - 2C_s), \\
M_s^{1*} &= M_s^{2*} = \frac{3}{2} C_s + \frac{1}{4}(\pi - u), \\
n_b^{1*} &= n_b^{2*} = \frac{1}{2} \quad \text{and} \quad n_s^{1*} = n_s^{2*} = \frac{1}{4(\gamma + \tau_s)} (u + \pi - 2C_s), \\
\nu_s^a &= \frac{\tau_s}{4(\gamma + \tau_s)} (u + \pi - 2C_s), \\
\nu_b^a &= \frac{1}{4(\gamma + \tau_s)} (u^2 + 4\pi u + \pi^2 - 2(u + \pi) C_s) - \tau_b - C_b.
\end{align*}
\]

Repeating the analysis for interaction on open platforms, we find:

\[
\begin{align*}
n_b^1 &= n_b^2 = \frac{1}{2} \quad \text{and} \quad n_s^1 = n_s^2 = \frac{1}{4(\gamma + \tau_s)} \pi, \\
\nu_s^o &= \frac{\tau_s}{2(\gamma + \tau_s)} \pi \quad \text{and} \quad \nu_b^o = \frac{1}{2(\gamma + \tau_s)} \pi u.
\end{align*}
\]
Considering sellers’ incentives to innovate, we compute:

\[ \Delta^m = \begin{cases} \frac{\gamma s}{2(\gamma + \tau_s)} (u(y') - u(y) + \pi(y') - \pi(y)) & \text{(multihoming sellers)}, \\ \frac{\gamma s}{4(\gamma + \tau_s)} (u(y') - u(y) + \pi(y') - \pi(y)) & \text{(singlehoming sellers)}. \end{cases} \]

\[ \Delta^n = \begin{cases} \frac{\gamma s}{(\gamma + \tau_s)} (\pi(y') - \pi(y)) & \text{(multihoming sellers)}, \\ \frac{\gamma s}{2(\gamma + \tau_s)} (\pi(y') - \pi(y)) & \text{(singlehoming sellers)}. \end{cases} \]

Comparing the expressions for \( \Delta^m \) and \( \Delta^n \) (focusing on sellers who multihome or singlehome in the two cases), we have that \( \Delta^m > \Delta^n \)

\[ \iff \frac{\gamma s}{2(\gamma + \tau_s)} (u(y') - u(y) + \pi(y') - \pi(y)) > \frac{\gamma s}{(\gamma + \tau_s)} (\pi(y') - \pi(y)) \]

\[ \iff u(y') - u(y) > \pi(y') - \pi(y), \]

which is the same condition as in Proposition 2.

**Competitive bottlenecks when buyers multihome.** We proceed as in Section 5 and find the following expressions under the new specification:

\[ n_s^i = \frac{1}{2} + \frac{\pi(M_b^i - M_s^i) + \tau_b(M_s^i - \Gamma_s)}{2(\tau_b (\gamma + \tau_s) - u \pi)}, \]

\[ n_b^i = \frac{u}{\tau_b} \left( \frac{1}{2} + \frac{\pi(M_b^i - M_s^i) + \tau_b(M_s^i - \Gamma_s)}{2(\tau_b (\gamma + \tau_s) - u \pi)} \right) - \frac{M_b^i}{\tau_b}, \]

\[ M_s^{1*} = M_s^{2*} = \gamma + \tau_s + C_s - \frac{u}{4\tau_b} (3\pi + u - 2C_b), \]

\[ M_b^{1*} = M_b^{2*} = \frac{1}{2} C_b + \frac{1}{4} (u - \pi), \]

\[ n_s^{1*} = n_s^{2*} = \frac{1}{2} \text{ and } n_b^{1*} = n_b^{2*} = \frac{1}{4\tau_b} (u + \pi - 2C_b), \]

\[ v_s^* = \frac{1}{4\tau_b} \left( u^2 + 4\pi u + \pi^2 - 2(\pi + u) C_b - 2\gamma \tau_b \right) - (C_s + \tau_s + \gamma), \]

\[ v_b^* = \frac{1}{4} \left( u + \pi - 2C_b \right). \]

If interaction takes place on open platform, we have:

\[ n_s^1 = n_s^2 = \frac{1}{2} \text{ and } n_b^1 = n_b^2 = \frac{1}{2\tau_b} u, \]

\[ v_s^m = \frac{1}{2\tau_b} u \pi - \gamma \frac{1}{2} \text{ and } v_b^m = \frac{1}{2} u. \]

Writing \((u_0, \pi_0)\) for \((u(y), \pi(y))\) and \((u_1, \pi_1)\) for \((u(y'), \pi(y'))\), we compute

\[ \Delta^m = \frac{1}{4\tau_b} \left( u_0^2 + 4\pi_1 u_1 + \pi_1^2 - 2(\pi_1 + u_1) C_b - (u_0^2 + 4\pi_0 u_0 + \pi_0^2 - 2(\pi_0 + u_0) C_b) \right), \]

\[ \Delta^n = \frac{1}{4\tau_b} (u_1 \pi_1 - u_0 \pi_0). \]
It follows that

\[ \Delta^m - \Delta^n = \frac{1}{2\tau_b} \left( u_1 + \pi_0 + \pi_1 + u_0 - 2C_b \right) \left( u_1 - u_0 - \pi_0 + \pi_1 \right), \]

which is positive as long as \( u_1 + \pi_1 > u_0 + \pi_0 \) or \( u(y') - u(y) + \pi(y') - \pi(y) > 0 \). This is the same condition as in Proposition 3.

**Appendix 2: Unique and stable equilibrium at stage 2**

We show here that the equilibrium we derived at stage 2 of the two-sided singlehoming game is unique and stable. From the identification of the buyer and the seller who are indifferent between the two platforms, we defined the number of buyers visiting platform \( i \) as a function of the number of sellers visiting this platform, and vice versa:

\[
\begin{align*}
&n_b^i(n_s^i) = \frac{1}{2} + \frac{1}{2\tau_b} \left[ (2n_s^i - 1)u - (M_b^i - M_b^j) \right], \\
n_s^i(n_b^j) = \frac{1}{2} + \frac{1}{2\tau_s} \left[ (2n_b^i - 1)\pi - (M_s^i - M_s^j) \right].
\end{align*}
\]

The solution to this system of equation was given by expressions (5) and (6):

\[
\begin{align*}
\hat{n}_b^i & = \frac{1}{2} + \frac{u(M_b^j - M_b^i) + \tau_s(M_b^j - M_b^i)}{2(\tau_b\tau_s - u\pi)}, \\
\hat{n}_s^i & = \frac{1}{2} + \frac{\pi(M_b^j - M_b^i) + \tau_b(M_s^j - M_s^i)}{2(\tau_b\tau_s - u\pi)}.
\end{align*}
\]

Assuming that \( \tau_b\tau_s > u\pi \), we have that \( n_b^i, n_s^i > 0 \). To have an interior solution, we further need \( n_b^i, n_s^i < 1 \). This is so provided that

\[
\begin{align*}
&u(M_s^j - M_s^i) + \tau_s(M_b^j - M_b^i) < \tau_b\tau_s - u\pi, \\
&\pi(M_b^j - M_b^i) + \tau_b(M_s^j - M_s^i) < \tau_b\tau_s - u\pi.
\end{align*}
\]

**Uniqueness.** Let us examine if there could be another equilibrium under these conditions. The only candidates are the situations where all agents on one and/or on the other side of the market concentrate on the same platform. Consider the situation where all buyers and all sellers visit platform 1. For this to be an equilibrium, the buyer and the seller located at 1 must prefer platform 1 over platform 2, expecting that all agents will be on platform 1. That is, the following two conditions must be met: (B) \( v_b^1 - \tau_b \geq v_b^2 \Leftrightarrow u \geq M_b^1 - M_b^2 + \tau_b \), and (S) \( v_s^1 - \tau_s \geq v_s^2 \Leftrightarrow \pi \geq M_s^1 - M_s^2 + \tau_s \). Multiplying both sides of condition (S) by \( u \), we get

\[ \pi u \geq \pi u \geq M_s^1 - M_s^2 + \tau_s \]

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\( u\pi \geq u(M_s^1 - M_s^2) + u\tau_s; \) using condition (B), we further have that \( u\pi \geq u(M_s^1 - M_s^2) + \tau_s(M_b^1 - M_b^2) + \tau_0\tau_s, \) which can be rewritten as \( \tau_0\tau_s - u\pi \leq u(M_s^1 - M_s^2) + \tau_s(M_b^1 - M_b^2). \) It is easily seen that the latter condition contradicts the first condition in (20). This proves that we cannot have an equilibrium where all agents concentrate on the same platform when the conditions in (20) are satisfied. Similar arguments can be used to show that conditions (20) also exclude situations where only one side of the market concentrates on the same platform.

**Stability.** We show now that expressions (5) and (6) also define a stable market configuration. Graphically, in a \((n_s, n_b)\) plane, the line \( n_s(n_b) \) must cross the line \( n_b(n_s) \) from below. Analytically, following Hagiu (2006b), we justify this by postulating a dynamic adjustment process for fixed \((M_b^i, M_s^i, M_b^j, M_s^j)\) of the following type: starting from any \((n_s^i(0), n_b^i(0))\), the market configuration \((n_s^i(t), n_b^i(t))\) evolves according to

\[
\begin{pmatrix}
\dot{n}_s^i(t) \\
\dot{n}_b^i(t)
\end{pmatrix} = \begin{pmatrix}
E \left( \frac{1}{2} + \frac{1}{2\tau_s} \left[ (2n_b^i(t) - 1)\pi - (M_b^i - M_s^i) \right] \right) \\
F \left( \frac{1}{2} + \frac{1}{2\tau_s} \left[ (2n_s^i(t) - 1)\pi - (M_b^j - M_s^j) \right] \right)
\end{pmatrix}
\]

where \( E \) and \( F \) are two positive constants. Figure ?? depicts the phase diagram of this process (in the case where \( M_b^1 = M_b^2 \) and \( M_s^1 = M_s^2 \)). It is easily seen that the assumption \( \tau_b\tau_s > u\pi \) guarantees that the process converges to \( (\hat{n}_s^i, \hat{n}_b^i) \) defined by expressions (5) and (6).

**Appendix 3: Competitive bottlenecks and decentralized investment decisions**

We repeat here, for the case of competitive bottlenecks when seller multihome, the analysis that we performed in Subsection 3.3.\(^{24}\) Recall that there is initially an independently drawn measure \( \mu \) of sellers who have invested. Seller and buyer surplus gross of any opportunity cost of visiting a platform \( i \) are written as

\[
\begin{align*}
\upsilon_s^i &= \begin{cases} 
  n_s^i\pi' - M_s^i & \text{if seller has invested,} \\
  n_s^i\pi - M_s^i & \text{if seller has not invested,}
\end{cases} \\
\upsilon_b^i &= n_b^i(\mu u' + (1 - \mu) u) - M_b^i,
\end{align*}
\]

\(^{24}\)The details of the computations can be obtained from the authors upon request.
where $\pi' \equiv \pi(y')$ and $\pi \equiv \pi(y)$ denote the net gain from trade for, respectively, a seller that has invested and a seller that has not, while $u' \equiv u(y')$ and $u \equiv u(y)$ denote the net gain from trade for a buyer if the buyer interacts, respectively, with a seller who has invested or with a seller who has not.

Suppose first that platforms cannot price discriminate between investing and non-investing sellers. In that case, the per platform seller equilibrium surplus can be found as

$$v^*_s = \begin{cases} 
\frac{1}{2} \pi' - \frac{1}{4} C_s - \frac{1}{4} (1 - \mu) (\pi - u) - \frac{1}{2} \mu (\pi' - u') & \text{if the seller has invested}, \\
\frac{1}{2} \pi - \frac{1}{2} C_s - \frac{1}{4} (1 - \mu) (\pi - u) - \frac{1}{4} \mu (\pi' - u') & \text{if the seller has not invested}.
\end{cases}$$

Let us now compute incentives to innovate. With for-profit platforms, we compute the change of surplus for a firm that was not investing beforehand but that now decides to invest along with a randomly drawn measure $\delta$ of other sellers. Accordingly, the incentive to innovate for a singlehoming seller is computed as

$$\Delta^m = \left[ \frac{1}{2} \pi' - \frac{1}{4} C_s - \frac{1}{4} (1 - \mu - \delta) (\pi - u) - \frac{1}{4} \mu (\pi' - u') \right] - \left[ \frac{1}{2} \pi - \frac{1}{2} C_s - \frac{1}{4} (1 - \mu) (\pi - u) - \frac{1}{4} \mu (\pi' - u') \right] = \frac{1}{2} (\pi' - \pi) + \frac{1}{4} \delta (\pi - u - \pi' + u').$$

As for the case of free platforms, the analysis is left unchanged and the the incentive to innovate for a singlehoming seller is given by $\Delta^n = \frac{1}{2} (\pi' - \pi)$. Comparing the latter two expressions, we conclude that incentives to innovate are larger under intermediated trade as long as $\delta > 0$ and $\pi - u - \pi' + u' > 0 \Leftrightarrow u' - u > \pi' - \pi$, which is equivalent to condition (18).

If platforms are able to price discriminate, we find the following per platform seller equilibrium surplus:

$$v^*_s = \begin{cases} 
\frac{1}{4} (u' + \pi') - \frac{1}{2} C_s & \text{if the seller has invested}, \\
\frac{1}{4} (u + \pi) - \frac{1}{2} C_s & \text{if the seller has not invested}.
\end{cases}$$

We then compute the incentive to innovate as the difference between the surplus of an investing seller and of a non-investing seller, i.e.,

$$\Delta^m = \left[ \frac{1}{4} (u' + \pi') - \frac{1}{2} C_s \right] - \left[ \frac{1}{4} (u + \pi) - \frac{1}{2} C_s \right] = \frac{1}{4} (u' + \pi' - u - \pi).$$

We observe that $\Delta^m > \Delta^n = \frac{1}{2} (\pi' - \pi) \Leftrightarrow u' - u > \pi' - \pi$, which is again equivalent to condition (18).
References


